

notes the overall mean of the data.

The "unexplained variance", or "within-group variability" is

$$\sum_{ij} (Y_{ij} - \bar{Y}_{i.})^2 / (N - K),$$

where  $Y_{ij}$  is the  $j^{\text{th}}$  observation in the  $i^{\text{th}}$  out of  $K$  groups and  $N$  is the overall sample size. This F-statistic follows the F-distribution with  $K - 1, N - K$  degrees of freedom under the null hypothesis. The statistic will be large if the between-group variability is large relative to the within-group variability, which is unlikely to happen if the population means of the groups all have the same value.

Note that when there are only two groups for the one-way ANOVA F-test,  $F = t^2$  where  $t$  is the Student's  $t$  statistic.

## Regression problems

Consider two models, 1 and 2, where model 1 is 'nested' within model 2. Model 1 is the Restricted model, and model two is the Unrestricted one. That is, model 1 has  $p_1$  parameters, and model 2 has  $p_2$  parameters, where  $p_2 > p_1$ , and for any choice of parameters in model 1, the same regression curve can be achieved by some choice of the parameters of model 2. (We use the convention that any constant parameter in a model is included when counting the parameters. For instance, the simple linear model  $y = mx + b$  has  $p = 2$  under this convention.) The model with more parameters will always be able to fit the data at least as well as the model with fewer parameters. Thus typically model 2 will give a better (i.e. lower error) fit to the data than model 1. But one often wants to determine whether model 2 gives a *significantly* better fit to the data. One approach to this problem is to use an  $F$  test.

If there are  $n$  data points to estimate parameters of both models from, then one can calculate the  $F$  statistic (coefficient of determination), given by

$$F = \frac{\left( \frac{RSS_1 - RSS_2}{p_2 - p_1} \right)}{\left( \frac{RSS_2}{n - p_2} \right)}$$

facweb

for table

<http://facweb.cs.depaul.edu/~sjost/csc425/documents/stat-tables/stat-tables.pdf>

where  $RSS_i$  is the residual sum of squares of model  $i$ . If your regression model has been calculated with weights, then replace  $RSS_i$  with  $\chi^2$ , the weighted sum of squared residuals. Under the null hypothesis that model 2 does not provide a significantly better fit than model 1,  $F$  will have an  $F$  distribution, with  $(p_2 - p_1, n - p_2)$  degrees of freedom. The null hypothesis is rejected if the  $F$  calculated from the data is greater than the critical value of the  $F$  distribution for some desired false-rejection probability (e.g. 0.05). The F-test is a Wald test.

## One-way ANOVA example