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# Measurement of Magnetic Susceptibilities and the Adoption of SI Units

**SI** is an abbreviation in many languages for Système International d'Unités; it is an extension and refinement of the traditional metric system. It is intended in this article to show how the adoption of SI units affects the calculations required for the determination of magnetic susceptibility. Since most of the existing information on magnetic susceptibilities is in CGS units it is important to be able to convert from existing values to the corresponding values in SI, both when making the correction for diamagnetic properties and for comparative purposes when interpreting results.

In most chemistry texts magnetic susceptibility is derived from the expression

$$B = H + 4\pi I \qquad (1)$$

which is the CGS expression relating field intensity within a substance ($B$), and the applied field ($H$). Note that the permeability of a vacuum ($\mu_0$) does not enter into this expression, since it is a dimensionless quantity having the value unity. In the MKSA system the relation between field intensity and applied field is given by

$$\frac{B}{\mu_0} = H + M \qquad (2)$$

$\mu_0$ does not have the value unity and has dimension (length)(mass)(time)$^{-2}$(current)$^{-2}$. This expression must be used for further derivation when SI units[1] are adopted. $B$ is the magnetic flux density, Webers per square meter (Wb m$^{-2}$); $H$, the magnetic field strength, amperes per meter (A m$^{-1}$); $\mu_0$, the permeability of a vacuum $= 4\pi \times 10^{-7}$ Henrys per meter (Hm$^{-1}$); and $M$, the magnetization, A m$^{-1}$. Dividing eqn. (2) throughout by $H$ gives

$$\frac{B}{\mu_0 H} = 1 + \frac{M}{H}$$

The quotient $M/H$, known as the volume susceptibility, is given the symbol $\chi$, and is dimensionless.

In the CGS system the relation and symbolism are different, and from eqn. (1)

$$\frac{B}{H} = 1 + 4\pi I/H = 1 + 4\pi K$$

where $B$ is the magnetic flux density, gauss; $H$, the magnetic field strength, oersteds; $I$, the intensity of magnetization, gauss; and $K$, the volume susceptibility, dimensionless. If $B = x$ gauss and $H = y$ oersteds then on the CGS

$$1 + 4\pi K = x/y$$

On the SI

$$x \text{ gauss} = x \times 10^{-4} \text{ Wb m}^{-2}$$

$$y \text{ oersteds} = y \times \frac{1}{4\pi} \times 10^3 \text{ A m}^{-1}$$

$$\therefore \frac{B}{\mu_0 H} = \frac{(x \times 10^{-4})}{(4\pi \times 10^{-7}) \times (y \times \frac{1}{4\pi} \times 10^3)} = \frac{x}{y} = 1 + \chi$$

Comparing the two systems

$$1 + \chi = \frac{x}{y} = 1 + 4\pi K$$

$$\therefore \chi = 4\pi K$$

Other expressions in use are the mass susceptibility $\chi_m$ and the molar susceptibility $\chi_M$, which are related to $\chi$ in the following way using SI

$$\chi_m = \frac{\chi}{\rho} \text{ m}^3 \text{ kg}^{-1}$$

$$\chi_M = \frac{10^{-3}\chi M}{\rho} \text{ m}^3 \text{ mol}^{-1}$$

where $M$ is the molecular weight, g mol$^{-1}$, and $\rho$, the density, kg m$^{-3}$. The corresponding units on the CGS are ml g$^{-1}$ and ml mol$^{-1}$. To convert values of mass susceptibility on CGS to SI multiply by $4\pi \times 10^{-3}$, and to convert values of molar susceptibility from CGS to SI multiply by $4\pi \times 10^{-6}$.

In an experiment on the Gouy balance a sample of tris(acetylacetonato)manganese(III) (1.4370 g) showed an apparent increase in mass of 0.0252 g. The applied field was 4000 oersteds and the cross-sectional area of the sample was 0.18 cm$^2$. Volume of the sample = 2.546 ml. Measurements were made at 295°K. On the SI the relation between increase in mass and volume susceptibility is

$$g\Delta w = \tfrac{1}{2}\mu_0(\chi - \chi_0)H^2 A$$

where $g$ is the acceleration due to gravity, 9.81 m s$^{-2}$; $\Delta w$, the apparent increase in mass, $0.0252 \times 10^{-3}$kg; $\mu_0$, the permeability of a vacuum, $4\pi \times 10^{-7}$ H m$^{-1}$; H, the applied field strength, $4000/(4\pi \times 10^{-3}) = 10^6/\pi$ A m$^{-1}$; A, the cross-sectional area of the sample, $0.18 \times 10^{-4}$ m$^2$; $\chi$, the volume susceptibility of the sample; and $\chi_0$, the volume susceptibility of air, $4\pi \times 0.029 \times 10^{-6}$. Substituting into the equation

$$\frac{2 \times 9.81 \times 0.0252 \times 10^{-3}}{\left(\frac{10^{12}}{\pi^2}\right) \times 0.18 \times 10^{-4} \times 4\pi \times 10^{-7}} + 4 \times 0.029 \times 10^{-6} = \chi$$

[1] McGlashan, M. L., "Physico-Chemical Quantities and Units" R.I.C. Monograph for Teachers No. 15, July 1968. This may be obtained from the Royal Institute of Chemistry, 30 Russell Square, London W.C. 1, England.

$$\therefore \chi = 216.1 \times 10^{-6}$$

The molar susceptibility

$$\chi_M = \frac{M \times \chi\, 10^{-3}}{\rho}$$

$M$ is the molecular weight, 351.9 g mol$^{-1}$; and $\rho$, the density of the sample, $(1.437 \times 10^{-3})/(2.546 \times 10^{-6}) = 0.564 \times 10^3$kg m$^{-3}$. Note that in SI the molecular weight is in gram not kilogram, e.g., 351.9 g mol$^{-1}$ *not* 351.9 kg mol$^{-1}$.

$$\therefore \chi_M = \frac{216.1 \times 10^{-6} \times 351.9 \times 10^{-3}}{0.564 \times 10^3}$$

$$\chi_M = 134800 \times 10^{-12}\ \text{m}^3\ \text{mol}^{-1}$$

The molar susceptibility has to be corrected for diamagnetic constituents. Inevitably the diamagnetic corrections in the existing literature are given in CGS units. To convert to SI multiply by $4\pi \times 10^{-6}$.

| | | | |
|---|---|---|---|
| $Mn^{3+}$ | $1 \times 40\pi \times 10^{-12}$ | = | 125.7 |
| C | $15 \times 24\pi \times 10^{-12}$ | = | 1131.3 |
| H | $21 \times 11.72\pi \times 10^{-12}$ | = | 673.5 |
| O | $6 \times -6.92\pi \times 10^{-12}$ | = | $-130.5$ |
| | Diamagnetic correction | = | $1800 \times 10^{-12}$ m$^3$ mol$^{-1}$ |

The corrected value of the molar susceptibility $\chi_M^{corr} = (134800 + 1800) \times 10^{-12}$

$$\chi_M^{corr} = 136600 \times 10^{-12}\ \text{m}^3\ \text{mol}^{-1}$$

Molar susceptibility and magnetic moment are related by

$$\chi_M = \frac{\mu_0 N \mu^2}{3kT}$$

where $\mu_0$ is the permeability of a vacuum $= 4\pi \times 10^{-7}$ H m$^{-1}$; $N$, the Avogadro constant $= 6.023 \times 10^{23}$ mol$^{-1}$; $k$, the Boltzmann constant $= 1.381 \times 10^{23}$J K$^{-1}$; $T$, the temperature $= 295°$K; and $\mu$, the magnetic moment, A m$^2$. Substituting into the equation

$$\mu^2 = \frac{1.366 \times 10^{-7} \times 3 \times 1.381 \times 10^{-23} \times 295}{6.023 \times 10^{23} \times 4\pi \times 10^{-7}}$$

$$= 22.06 \times 10^{-46}$$

$$\therefore \mu = 4.69 \times 10^{-23}\text{A m}^2$$

If the magnetic moment is expressed in Bohr magnetons where 1 Bohr magneton $= 9.27 \times 10^{-24}$ A m$^2$ then

$$\mu = 46.9/9.27 = 5.06\ \text{BM}$$

On the CGS system the expression

$$g\Delta w = {}^1/_2(K - K_0)H^2A$$

would be used for such a calculation, when using the data quoted above

$$\chi_M = 10730 \times 10^{-6}\ \text{ml mol}^{-1}$$

Similarly the magnetic moment calculated using the CGS expression

$$\chi_M = \frac{N\mu^2}{3kT}$$

returns a value of $\mu = 4.69 \times 10^{-20}$ erg gauss$^{-1}$ or 5.06 Bohr magnetons.

Note that in the CGS system, using CGS values of physical constants

$$1\ \text{Bohr magneton} = \frac{eh}{4\pi m} = 9.27 \times 10^{-21}\ \text{erg gauss}^{-1}$$

whereas on the SI, using SI values for physical constants

$$1\ \text{Bohr magneton} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\text{A m}^2$$

The spin only formula $\mu = 4S(S + 1)$ BM where $S$ is the total spin quantum number, can be applied to the values for $\mu$ obtained on the SI and the CGS. Since the values for $\mu$ in Bohr magnetons are the same in both systems, it follows that the total spin quantum number, i.e., number of unpaired electrons, is independent of the system of units used.